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Dilaton and axion bremsstrahlung from collisions of cosmic (super)strings

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Abstract
We calculate dilaton and axion radiation generated in the collision of two straight initially unexcited strings and give a rough cosmological estimate of dilaton and axion densities produced via this mechanism in the early universe.

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1. Introduction

Recently the early universe models involving strings and branes moving in higher-dimensional spacetimes received renewed attention [1–6]. In particular, the problem of the dimensionality of spacetime can be explored within the brane gas scenario [1–3]. Another new suggestion is the possibility of cosmic superstrings with lower tension than those in the field-theoretical GUT strings [3]. Superstrings as cosmic strings candidates stimulate reconsideration of the cosmic string evolution taking into account new features such as the existence of the dilaton and antisymmetric form fields and extra dimensions. The main role in this evolution is played by radiation processes. The radiation mechanism which has been mostly studied in the past consists in the formation of the excited closed loops which subsequently loose their excitation energy emitting gravitons [7], axions [8] and dilatons [9–12].

In this paper we consider the bremsstrahlung mechanism of string radiation [13] which works for initially unexcited strings undergoing a collision. This effect is similar to bremsstrahlung under collision of point charges in electrodynamics. In the perturbation expansion in terms of the fine structure constant, bremsstrahlung is the second-order process. In the case of strings we develop a classical perturbation scheme for two endless unexcited long strings which move one with respect to another in two parallel planes being inclined at an angle. It was shown earlier that in four spacetime dimensions there is no gravitational bremsstrahlung under collision of straight strings [13]. This can be traced to the absence of gravitons in $1+2$ gravity. It is not a coincidence that in four dimensions there is no gravitational renormalization of the string tension either [15]. But there is no such dimensional argument

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in the case of the axion field there such dimensional argument and it was demonstrated that string bremsstrahlung takes place indeed [14] within the model in flat space. Here we extend this result to the full gravitating case including also the dilaton field. Strings interacts via the dilaton, axion and graviton exchange. Radiation arises in the second-order approximation in the coupling constants provided the (projected) intersection point moves with superluminal velocity. Thus, the string bremsstrahlung can be viewed as a manifestation of the Cherenkov effect.

2. String interactions

Consider a pair of relativistic strings,

$$x^\mu = x_n^\mu(\sigma_n^a), \qquad \mu = 0, 1, 2, 3, \quad \sigma_a = (\tau, \sigma), \quad a = 0, 1,$$

where $n = 1, 2$ is the index labelling the two strings. The four-dimensional spacetime metric signature $+, - - -$ and $(+, -)$ for the string world-sheets metric signature. Strings interact via the gravitational $g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$, dilatonic $\phi(x)$ and axion (Kalb-Ramond) field $B_{\mu\nu}(x)$:

$$S = -\sum_n \int \left\{ \frac{\mu}{2} \partial_a x_n^\mu \partial_b x_n^\nu g_{\mu\nu} \gamma^{ab} \sqrt{-\gamma}\, \mathrm{e}^{2\alpha_n \phi} + 2\pi f \partial_a x_n^\mu \partial_b x_n^\nu \epsilon^{ab} B_{\mu\nu} \right\} \mathrm{d}^2\sigma \\ + \int \left\{ 2\partial_\mu \phi \partial_\nu \phi g^{\mu\nu} + \frac{1}{6} H_{\mu\nu\rho} H^{\mu\nu\rho}\, \mathrm{e}^{-4\alpha\phi} - \frac{R}{16\pi G} \right\} \sqrt{-g}\, \mathrm{d}^4 x. \qquad (1)$$

Here μ_n are the (bare) string tension parameters, α and f are the corresponding coupling parameters, $\epsilon^{01} = 1, \gamma_{ab}$ is the induced metric on the world-sheets. In what follows, we linearize the dilaton exponent as $\mathrm{e}^{2\alpha\phi} \simeq 1 + 2\alpha\phi$.

The totally antisymmetric axion field strength is defined as

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}. \qquad (2)$$

Variation of the action (1) over x_n^μ leads to the equations of motion for strings,

$$\partial_a \left(\mu \partial_b x_n^\nu g_{\mu\nu} \gamma^{ab} \sqrt{-\gamma}\, \mathrm{e}^{2\alpha\phi} + 4\pi f \partial_b x_n^\nu \epsilon^{ab} B_{\mu\nu} \right) - \mu\alpha \partial_a x_n^\alpha \partial_b x_n^\beta g_{\alpha\beta} \gamma^{ab} \sqrt{-\gamma}\, \mathrm{e}^{2\alpha\phi} \partial_\mu \phi \\ - \frac{\mu}{2} \partial_a x_n^\alpha \partial_b x_n^\beta \gamma^{ab} \sqrt{-\gamma}\, \mathrm{e}^{2\alpha\phi} \partial_\mu g_{\alpha\beta} = 0. \qquad (3)$$

Variation with respect to the field variables ϕ, $B_{\mu\nu}$ and $g_{\mu\nu}$ leads to the dilaton equation

$$\partial_\mu (g^{\mu\nu} \partial_\nu \phi \sqrt{-g}) + \frac{\alpha}{6} H^2\, \mathrm{e}^{-4\alpha\phi} + \frac{\mu\alpha}{4} \int \partial_a x_n^\mu \partial_b x_n^\mu g_{\mu\nu} \gamma^{ab}\, \mathrm{e}^{2\alpha\phi} \delta^4(x - x_n(\sigma_n))\, \mathrm{d}^2\sigma = 0, \qquad (4)$$

the axion equation

$$\partial_\mu (H^{\mu\nu\lambda}\, \mathrm{e}^{-4\alpha\phi} \sqrt{-g}) + 2\pi f \int \partial_a x_n^\nu \partial_b x_n^\lambda \epsilon^{ab} \delta^4(x - x_n(\sigma_n))\, \mathrm{d}^2\sigma = 0, \qquad (5)$$

and the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left(\overset{\phi}{T}_{\mu\nu} + \overset{B}{T}_{\mu\nu} + \overset{st}{T}_{\mu\nu} \right), \\ \overset{st}{T}_{\mu\nu} = \sum \mu \int \partial_a x_{\mu n} \partial_b x_{\nu n} \gamma^{ab} \sqrt{-\gamma}\, \mathrm{e}^{2\alpha\phi} \frac{\delta^4(x - x_n(\sigma_n))}{\sqrt{-g}}\, \mathrm{d}^2\sigma, \qquad (6)$$

$$\overset{\phi}{T}_{\mu\nu} = 4 \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla\phi)^2 \right), \qquad \overset{B}{T}_{\mu\nu} = \left(H_{\mu\alpha\beta} H_\nu^{\alpha\beta} - \frac{1}{6} H^2 g_{\mu\nu} \right) \mathrm{e}^{-4\alpha\phi}. \qquad (7)$$

The constraint equations for each string read

$$\left(\partial_a x^\mu \partial_b x^\nu - \tfrac{1}{2} \gamma_{ab} \gamma^{cd} \partial_c x^\mu \partial_d x^\nu \right) g_{\mu\nu} = 0. \qquad (8)$$

Our calculation follows the approach of [13, 14] and consists in constructing solutions of the string equations of motion and dilaton, axion and graviton iteratively using the coupling constants α, f, G as expansion parameters.

Denote as $\overset{0}{x}{}^{\mu}$ the embedding function of the non-exited straight string stretched along the direction Σ^{μ} and moving as a whole with the four-velocity u^{μ}. This is the linear function of τ, σ:

$$\overset{0}{x}{}^{\mu} = d^{\mu} + u^{\mu}\tau + \Sigma^{\mu}\sigma, \tag{9}$$

where the constant vector d^{μ} can be regarded as an impact parameter. The zero-order spacetime metric is assumed flat $\overset{0}{g}_{\mu\nu} = \eta_{\mu\nu}$, and the zero-order world-sheet induced metric can be also made Minkowskian. Indeed, assuming $\eta_{\mu\nu}x^{\mu}_{a}x^{\nu b} = \delta^{a}_{b}$, i.e. $\eta_{\mu\nu}\Sigma^{\mu}\Sigma^{\nu} = -1$, $\eta_{\mu\nu}u^{\mu}u^{\nu} = 1$, $\eta_{\mu\nu}\Sigma^{\mu}u^{\nu} = 0$, one has $\overset{0}{\gamma}_{ab} = \partial_a \overset{0}{X}{}^{\mu}\partial_b \overset{0}{X}{}^{\nu}\eta_{\mu\nu} = \eta_{ab} = \mathrm{diag}(1, -1)$. Assuming that zero-order (external) dilaton and axion fields are absent $\overset{0}{\phi} = 0$, $\overset{0}{B}_{\mu\nu} = 0$, we expand all the field variables $\phi, B_{\mu\nu}, h_{\mu\nu}$ starting with the first order: $\phi = \overset{1}{\phi} + \overset{2}{\phi} + \cdots$, $B_{\mu\nu} = \overset{1}{B}_{\mu\nu} + \overset{2}{B}_{\mu\nu} + \cdots$, $h_{\mu\nu} = \overset{1}{h}_{\mu\nu} + \overset{2}{h}_{\mu\nu} + \cdots$.

The total dilaton, axion and graviton fields are the sums due to contributions of two strings: $\phi = \phi_1 + \phi_2$, $B^{\mu\nu} = B^{\mu\nu}_1 + B^{\mu\nu}_2$, $h^{\mu\nu} = h^{\mu\nu}_1 + h^{\mu\nu}_2$. Since in the zero order the strings are moving freely (9), the first-order dilaton $\overset{1}{\phi}_n$, axion $\overset{1}{B}{}^{\mu\nu}_{n}$ and graviton variables $\overset{1}{h}{}^{\mu\nu}_{n}$ do not contain radiative components. Substituting them into the equation (3), we then obtain the first order deformations of the world-sheets $\overset{1}{x}{}^{\mu}$, which are naturally split into contributions due to dilaton, axion and graviton exchange:

$$\overset{1}{x}{}^{\mu}_{n} = \overset{1}{x}{}^{\mu}_{n(\phi)} + \overset{1}{x}{}^{\mu}_{n(B)} + \overset{1}{x}{}^{\mu}_{n(h)}. \tag{10}$$

The deformation due to the dilaton reads

$$\overset{1}{x}{}^{\mu}_{n(\phi)}(\tau, \sigma) = -\mathrm{i}\frac{\alpha^2\mu}{16\pi^2}\int \frac{\Delta_{n'} D^{\mu}_{n'n}\mathrm{e}^{-\mathrm{i}q(d_n + u_n\tau + \Sigma_n\sigma)}}{q^2((q\Sigma_1)^2 - (qu_1)^2)}\mathrm{d}^4 q, \tag{11}$$

where $\Delta_{n'} = \mathrm{e}^{\mathrm{i}qd_{n'}}\delta(qu_{n'})\delta(q\Sigma_{n'})$, $D^{\mu}_{n'n} = \overset{0}{U}_{n'}\ (q^{\mu}\overset{0}{U}_{n} + 2\Sigma^{\mu}_{n}(\Sigma_n q) - 2u^{\mu}_{n}(u_n q))$, $\overset{0}{U}_{n} = \eta^{\mu\nu}\ \overset{0}{U}{}^{\mu\nu}_{n}$, $\overset{0}{U}{}^{\mu\nu}_{n} = u^{\mu}_{n}u^{\nu}_{n} - \Sigma^{\mu}_{n}\Sigma^{\nu}_{n}$, $\overset{0}{U}_{n} = 2$. The corresponding axion contribution is

$$\overset{1}{x}{}^{\mu}_{n(B)} = -\mathrm{i}\frac{2f^2}{\mu}\int \frac{X^{\mu}_{n'n}\Delta_{n'}\,\mathrm{e}^{-\mathrm{i}q(d_n + u_n\tau + \Sigma_n\sigma)}}{q^2[(q\Sigma_1)^2 - (qu_1)^2]}\,\mathrm{d}^4 q, \tag{12}$$

where $X^{\mu}_{n'n} = q^{\mu}A_{n'n} + B_{n'n}\Sigma^{\mu}_{n'} + C_{n'n}u^{\mu}_{n'}$, $A_{n'n} = (u_n u_{n'})(\Sigma_n\Sigma_{n'}) - (\Sigma_n u_{n'})(u_n\Sigma_{n'})$, $B_{n'n} = (qu_n)(u_{n'}\Sigma_n) - (\Sigma_n q)(u_n u_{n'})$, $C_{nn'} = (u_n\Sigma_{n'})(\Sigma_n q) - (qu_n)(\Sigma_n\Sigma_{n'})$ and the gravitational contribution is

$$\overset{1}{x}{}^{\mu}_{n(h)} = -\mathrm{i}\frac{2}{\pi}G\mu\int \frac{Z^{\mu}_{n'n}\Delta_{n'}\,\mathrm{e}^{-\mathrm{i}q(d_n + u_n\tau + \Sigma_n\sigma)}}{q^2[(q\Sigma_1)^2 - (qu_1)^2]}\,\mathrm{d}^4 q, \tag{13}$$

where $Z^{\mu}_{n'n} = \left(\overset{0}{W}{}^{\alpha\beta}_{n'}q^{\mu} - 2\ \overset{0}{W}{}^{\mu\alpha}_{n'}q^{\beta}\right)\overset{0}{U}_{n\alpha\beta} - \left(q^{\chi}\ \overset{0}{W}{}^{\alpha\beta}_{n'} - 2q^{\alpha}\ \overset{0}{W}{}^{\beta\chi}_{n'}\right)\overset{0}{U}{}^{\mu\chi}_{n}\ \overset{0}{U}_{n\alpha\beta}$. Here $\overset{0}{W}{}^{\alpha\beta}_{n} = \overset{0}{U}{}^{\alpha\beta}_{n} - \frac{1}{2}\eta^{\alpha\beta}\ \overset{0}{U}_{n}$.

It can be checked that the quantities $D^{\mu}_{n'n}$, $X^{\mu}_{n'n}$ and $Z^{\mu}_{n'n}$ satisfy the conditions $D^{\mu}_{n'n}u_{n\mu} = D^{\mu}_{n'n}\Sigma_{n\mu} = 0$, $X^{\mu}_{n'n}u_{n\mu} = X^{\mu}_{n'n}\Sigma_{n\mu} = 0$, $Z^{\mu}_{n'n}u_{n\mu} = Z^{\mu}_{n'n}\Sigma_{n\mu} = 0$, which ensure the fulfilment of the constraint equations (8) up to the first-order terms.

Radiation arises in the second-order field terms $\overset{2}{\phi}_n$ and $\overset{2}{B}{}_n^{\mu\nu}$ which are generated by the first-order currents $\overset{1}{J}_{(\phi)}$, $\overset{1}{J}{}_{(B)}^{\mu\nu}$ in the dilaton and axion field equations (4) and (5). These currents are constructed using the first-order quantities, so the resulting equations read

$$\Box\, \overset{2}{\phi} = 4\pi\ \overset{1}{J}_{(\phi)}, \qquad \Box\, \overset{2}{B}{}^{\mu\nu} = 4\pi\, \overset{1}{J}{}_{(B)}^{\mu\nu}, \tag{14}$$

where

$$\begin{aligned}
\overset{1}{J}_{(\phi)} = \sum_{n\neq n'} \Bigg\{ & \frac{\alpha\mu}{8\pi}\int \mathrm{d}^2\sigma \left[\big(\overset{0}{\dot{x}}{}_n^{(\mu}\overset{1}{\dot{x}}{}_n^{\nu)} - \overset{0}{x}{}_n'^{(\mu}\overset{1}{x}{}_n'^{\nu)}\big)\eta_{\mu\nu} - \frac{1}{2}\,\overset{0}{U}_n \overset{1}{x}{}_n^{\lambda}\partial_\lambda \right] \delta^4\big(x - \overset{0}{x}_n(\sigma_n)\big) \\
& + \frac{\alpha^2\mu}{8\pi}\int \mathrm{d}^2\sigma\ \overset{0}{U}_n\ \overset{1}{\phi}_{n'} + \frac{\alpha\mu}{16\pi}\int \mathrm{d}^2\sigma\ \overset{0}{U}{}_n^{\mu\nu}\ \overset{1}{h}_{n'\mu\nu} \\
& - \frac{1}{4\pi}\partial^\mu\big((\partial^\nu\ \overset{1}{\phi}_n)\ \overset{1}{\psi}_{n'\mu\nu}\big) + \frac{\alpha}{24\pi}H_n^2 \Bigg\}, \\
\overset{1}{J}{}_{(B)}^{\mu\nu} = \sum_{n\neq n'} \Bigg\{ & f\int \mathrm{d}^2\sigma \left[\big(\overset{0}{\dot{x}}{}_n^{[\mu}\overset{1}{x}{}_n'^{\nu]} + \overset{1}{\dot{x}}{}_n^{[\mu}\overset{0}{x}{}_n'^{\nu]}\big) - \frac{1}{2}\ \overset{0}{V}{}_n^{\mu\nu}\ \overset{1}{x}{}_n^{\lambda}\partial_\lambda \right] \delta^4\big(x - \overset{0}{x}_n(\sigma_n)\big) \\
& + \frac{1}{8\pi}\Box\ \overset{1}{B}{}_n^{\mu\nu}\ \overset{1}{h}_{n'} + \frac{1}{8\pi}\ \overset{1}{H}{}_n^{\lambda\mu\nu}\partial_\lambda\ \overset{1}{h}_{n'} - \frac{\alpha}{\pi}\Box\ \overset{1}{B}{}_n^{\mu\nu}\ \overset{1}{\phi}_{n'} - \frac{\alpha}{\pi}\ \overset{1}{H}{}_n^{\lambda\mu\nu}\partial_\lambda\ \overset{1}{\phi}_{n'} \Bigg\}.
\end{aligned} \tag{15}$$

Here the brackets (), [] denote symmetrization and alternation over indices with the factor $1/2$, $\psi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ and the D'Alembert operator is $\Box = -\eta^{\mu\nu}\partial_\mu\partial_\nu$. The right-hand sides of the field equations contain the first-order field quantities

$$\overset{1}{\phi} = \frac{\alpha\mu}{8\pi^2}\int \frac{\mathrm{e}^{\mathrm{i}q_\lambda \overset{0}{x}{}_n^\lambda}\delta(qu_n)\delta(q\Sigma_n)}{q^2 + 2i\epsilon q^0}\,\mathrm{d}^4 q, \qquad \overset{1}{B}{}_n^{\mu\nu} = \frac{f}{2\pi}\int \frac{\mathrm{e}^{\mathrm{i}q_\lambda \overset{0}{x}{}_n^\lambda}\ \overset{0}{V}{}_n^{\mu\nu}\delta(qu_n)\delta(q\Sigma_n)}{q^2 + 2\mathrm{i}\epsilon q^0}\,\mathrm{d}^4 q$$

and

$$\overset{1}{h}_{\mu\nu} = \frac{4\mu G}{\pi}\int \frac{W_{\mu\nu}\,\mathrm{e}^{-\mathrm{i}q_\lambda \overset{0}{x}{}_n^\lambda}\delta(qu_n)\delta(q\Sigma_n)}{q^2 + 2i\epsilon q^0}\,\mathrm{d}^4 q, \tag{16}$$

where $\overset{0}{V}{}_n^{\mu\nu} = u_n^\mu\Sigma_n^\nu - u_n^\nu\Sigma_n^\mu$.

Note that gravitational radiation in four dimensions is absent [13], so we do not consider the second-order graviton equation. The dilaton and axion radiation power can be computed as the reaction work given by the half sum of the retarded and advanced fields upon the sources [14] and can be presented in the form

$$\begin{aligned}
P_{(\phi)}^{\mu} &= \frac{16}{\pi}\int k^\mu \frac{k^0}{|k^0|}|\overset{1}{J}_{(\phi)}(k)|^2\delta(k^2)\,\mathrm{d}^4 k, \\
P^{(B)\mu} &= \frac{1}{\pi}\int k^\mu \frac{k^0}{|k^0|}\left|\overset{1}{J}{}_{(B)}^{\alpha\beta}(k)\right|^2\delta(k^2)\,\mathrm{d}^4 k.
\end{aligned} \tag{17}$$

The final formula for the dilaton and axion bremsstrahlung from the collision of two global strings can be obtained analytically in the case of the ultrarelativistic collision with the Lorentz factor $\gamma = (1 - v^2)^{-1/2} \gg 1$. We assume the BPS condition for the coupling constants [15] $\alpha\mu = 2\sqrt{2\pi}f$. The main contribution to radiation turns out to come from the graviton exchange terms. The spectrum has an infrared divergence due to the logarithmic dependence of the string interaction potential on distance, so a cutoff length Δ has to be introduced:

$$\begin{aligned} P^{(\phi)} &= \frac{200}{3}\pi G^2\alpha^2\mu^4 L\kappa^5\big(f(y) + \tfrac{1}{25} f_1(y)\big), \\ P^{(B)} &= \frac{16\pi^3 G^2\mu^2 L f^2\kappa^5}{3}(f(y) - f_2(y)), \end{aligned} \tag{18}$$

where L is the length of the string, $y = \frac{d}{\gamma\kappa\Delta}$, $\kappa = \gamma\cos\alpha$, α is the strings inclination angle and

$$f(y) = 12\sqrt{\frac{y}{\pi}}\,{}_2F_2\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -y\right) - 3\ln(4y\mathrm{e}^C), \tag{19}$$

$$f_1(y) = (1 - \mathrm{erf}(\sqrt{y}))\left(\frac{8}{3}y^3 - 30y^2 + 114y + \frac{169}{2}\right) - \frac{\mathrm{e}^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 - \frac{94}{3}y + 131\right), \tag{20}$$

$$f_2(y) = (1 - \mathrm{erf}(\sqrt{y}))\left(\frac{8}{3}y^3 + 6y^2 - 6y - \frac{5}{2}\right) - \frac{\mathrm{e}^{-y}\sqrt{y}}{\pi}\left(\frac{8}{3}y^2 + \frac{14}{3}y - 7\right), \tag{21}$$

with F being the generalized hypergeometric function and C Euler's constant.

3. Cosmological estimate

The evolution of cosmic superstring networks was recently discussed in [16–21]. It was shown that cosmic superstrings share a number of properties of usual cosmic strings, but there are also differences which may lead to observational signatures. In particular, for usual cosmic strings the probability of the loop formation $\mathcal{P}$ is of the order of unity, whereas for F-strings $\mathcal{P}$ is $10^{-3} \leqslant \mathcal{P} \leqslant 1$ and D-strings $10^{-1} \leqslant \mathcal{P} \leqslant 1$. The cosmic superstring network has a scaling solution and the characteristic scale is proportional to the square root of the reconnection probability. The typical separation between two long strings is comparable to the horizon size, $\zeta(t) \simeq \sqrt{\mathcal{P}}t$. The numerical results show that the network of long strings will reach an energy density

$$\rho_s = \frac{\mu}{\mathcal{P}t^2}. \tag{22}$$

Consider the scattering of an ensemble of randomly oriented straight strings on a selected target string in the rest frame of the latter. Since the dependence of the string bremsstrahlung on the inclination angle α is smooth, we can use for a rough estimate the particular result obtained for the parallel strings ($\alpha = 0$) introducing an effective fraction ν of 'almost' parallel strings (roughly $1/3$). For N strings in the normalization cube $V = L^3$, we have to integrate the radiation energy released in the collision with the impact parameter $d = x$ over the plane perpendicular to the target string with the measure $N/L^2 \cdot 2\pi x\,\mathrm{d}x$. To find the radiation power per unit time we have to divide the integrand by the impact parameter. Multiplying this quantity by the total number of strings N to get the radiation energy released per unit time within the normalization volume, we obtain in the axion case:

$$Q_{\mathrm{brem}} = \int_0^L P^0\nu\frac{N}{L^2}\frac{N}{V}2\pi\,\mathrm{d}x, \tag{23}$$

where we can use the equations (18) for P^0. Taking into account that the string number density is related to the energy density (22) via

$$\frac{N}{V} = \frac{\rho_s}{\mu L}, \tag{24}$$

and assuming for a rough estimate $L \sim \Delta \sim t$, we obtain

$$Q^{(\phi)}_{\text{brem}} \simeq 800\pi^2 G^2 \alpha^2 \mu^4 \nu \gamma^5 \ln \gamma \frac{1}{\mathcal{P} t^3}, \qquad Q^{(B)}_{\text{brem}} \simeq 64\pi^4 G^2 \mu^2 \nu f^2 \gamma^5 \ln \gamma \frac{1}{\mathcal{P} t^3}. \tag{25}$$

Note that the realistic value of γ is of the order of unity, while our formulae were obtained in the $\gamma \gg 1$ approximation. Still we hope to give the correct order of magnitude estimate.

Now we can calculate the energy density of the bremsstrahlung dilaton and axions for the radiation dominated Universe as a function of time. Since dilatons and axions are massless at this stage, their energy density scales with the Hubble constant as H^{-4}, so we have the equation

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -4H\varepsilon + Q_{\text{brem}}, \tag{26}$$

where $H = \frac{1}{2t}$. From here we obtain for the energy density of the bremsstrahlung dilaton and axions at the moment $t > t_0$:

$$\varepsilon^{(\phi)} \simeq 800\pi^2 G^2 \alpha^2 \mu^4 \nu \gamma^5 \ln \gamma \frac{\ln(t/t_0)}{\mathcal{P} t^2}, \qquad \varepsilon^{(B)} \simeq 64\pi^4 G^2 \mu^2 \nu f^2 \gamma^5 \ln \gamma \frac{\ln(t/t_0)}{\mathcal{P} t^2}, \tag{27}$$

where t_0 is the initial time of the long string formation.

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